Ratner 定理之有效版本及其应用

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Ratner 定理（1990's）完整解决了 Raghunathan 提出的齐性空间中幂幺子群作用的测度和轨道闭包分类的一系列猜想、揭示了幂幺子群作用的神奇刚性性质，深度提升了数论和动力系统研究的数学审美观。借由 Dani 和 Margulis（1980's）发现的齐性动力系统与数论之间的深刻联系，Ratner 定理不仅使得很多数论中的著名公开问题迎刃而解，其证明思想更直接引领了多位动力系统 Fields 奖得主的主要研究工作。

根据 Ratner 定理，幂幺子群的轨道会趋于均匀分布，然而对其收敛速度却无定论，是一个困扰人们很久的公开问题。 对这类问题的回答统称为有效版本的 Ratner 定理，因其在数论中的广泛应用前景吸引了 Einsiedler-Margulis-Venkates, Green-Tao, Lindenstrauss-Margulis, Strombergsson 等诸多名家对其研究。杨磊博士最近与合作者（Allen、Beresnevich、Chow 和 Velani）在 3 维单位格组成的模空间（即 SL(3,R)/SL(3,Z)）上对于一类重要的幂幺轨道建立了有效版本的 Ratner 定理，并利用它证明了强化版本的 Littlewood 猜想对于平面上一条(满足一自然丢番图性质的）直线上几乎所有点成立。 他们的研究方法有别于传统的傅里叶分析和李群表示论的方法、以及 Ratner 定理的原始证明，是一种全新的动力系统方法。这是否能用来给出一般情况下 Ratner 定理之有效版本？值得期待。

齐性动力系统与数论缘起何时？Ratner 定理美在何处？有效版本的 Ratner 定理又意指何方？12 月 21 日下午 3 时，杨磊博士将与您分享他的感悟、视角和见解。

Effective Ratner's Theorem and its applications

YANG, Lei (Sichuan University)

Ratner's Theorem (1990's) completely solves Raghunathan's famous conjectures about the orbit closure and measure rigidity of the actions of unipotent subgroups on homongeneous spaces. This seminal work has many important applications to number theory (due to Dani and Margulis's fundamental works in 1980's on the connections between number theory and homogeneous dynamics) and its deep ideas have influenced several Fields medalists from dynamics.

Ratner's Theorem predicts that every unipotent orbit will tend to be equidistributed in a homogeneous subspace, but it doesn't tell the speed of convergence, which is a longstanding open problem in general and any result in this direction is called an effective Ratner's Theorem. The study of this topic will have

further impact in number theory and has attracted many top experts including Einsiedler-Margulis-Venkates, Green-Tao, Lindenstrauss-Margulis, Strombergsson, etc. In a recent ongoing work, Dr. Lei Yang (joint with Allen, Beresnevich, Chow and Velani) establish an effective Ratner's theorem for unipotent orbits in the moduli space of 3-dimensional unimodular lattices, and prove that a strengthened Littlewood's conjecture holds for almost every point on a planar straight line (satisfying a natural Diophantine condition). Their new dynamical method might contain some insights for general Effective Ratner's Theorem.

On Dec. 21 (15:00-16:00), Dr. YANG, Lei will share with you his understanding, opinion and insights on the connections between homogenous dynamics and number theory, Ratner's Theorem and its effective distribution results.